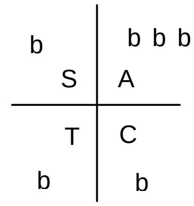
1. $\sin(x) - \sqrt{3}\cos(x) = k\sin(x-a)^0$
$= k\sin(x)\cos(a) - k\cos(x)\sin(a)$

equate coefficients



$\sin(x)$: $1 = k\cos(a)$

$\cos(x)$: $-\sqrt{3} = -k\sin(a)$

$k = \frac{1}{\cos(a)} = \frac{\sqrt{3}}{\sin(a)}$

$\frac{\sin(a)}{\cos(a)} = \frac{\sqrt{3}}{1}$ $\tan(a) = \sqrt{3}$ since the solution lies in quadrant 1, $a = 60^0$

$k = \sqrt{1+3} = 2$

So: $\sin(x) - \sqrt{3}\cos(x) = 2\sin(x - 60)^0$

To solve $f(x) = 1$

$2\sin(x - 60)^0 = 1$

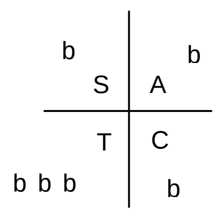
$\sin(x - 60)^0 = ½$

$(x - 60)^0 = 30, 150$

$x = 90^0, 210^0$

2. $\cos(x) - \sin(x) = k\sin(x-a)^0$
$= k\sin(x)\cos(a) - k\cos(x)\sin(a)$

equate coefficients



$\sin(x)$: $-1 = k\cos(a)$

$\cos(x)$: $1 = -k\sin(a)$

$k = \frac{-1}{\cos(a)} = \frac{-1}{\sin(a)}$

$\tan(a) = 1$ *since the solution lies in quadrant 3,* $a = \frac{5p}{4}$

$k = \sqrt{2}$

So: $\cos(x) - \sin(x) = \sqrt{2}\sin(x - \frac{5p}{4})$

To solve f(x) = -1

$\sqrt{2}\ sin(x - \frac{5p}{4}) = -1$

$sin(x - \frac{5p}{4}) = -\frac{1}{\sqrt{2}}$

$(x - \frac{5p}{4}) = -\frac{3p}{4}, -\frac{p}{4}$

$x = \frac{p}{2}, p$ *note that any other solutions will lie outside the domain*

3. *y = 3cos(x) + 4sin(x) + 5*

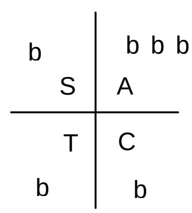
First express *3cos(x) + 4sin(x) in the form kcos(x - a)*^{0}

3cos(x) + 4sin(x) = kcos(x - a)^0
= kcos(x)cos(a) + ksin(x)sin(a)

equate coefficients

sin(x): *3 = kcos(a)*

cos(x) *4 = ksin(a)*



$k = \frac{3}{cos(a)} = \frac{4}{sin(a)}$

tan(a) = 4/3 *since the solution lies in quadrant 1, a= 53.1*^{0}

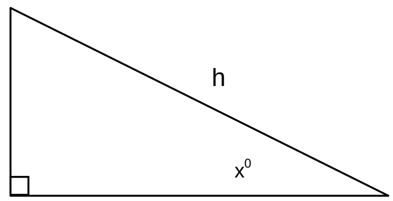
$k = \sqrt{9+16} = 5$

3cos(x) + 4sin(x) + 5 = 5cos(x – 53.1)^0 *+ 5*

maximum value = 5 + 5 = 10 occurs when (x – 53.1)^0 *= 0* *i.e. x = 53.1*^{0}

mimimum value = -5 + 5 = 0 occurs when (x – 53.1)^0 *= 180*^{0} *i.e x = 233.1*^{0}

4.



let the opposite side be a

sin(x) = a/h *a = hsin(x)*

let the adjacent side be b

cos(x) = b/h *b = hcos(x)*

The perimeter is: a + b + h = 20

substituting gives: *hsin(x) + hcos(x) + h = 20*

h(sin(x) + cos(x) + 1) = 20

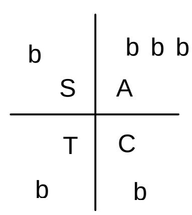
$$h = \frac{20}{\sin(x)+\cos(x)+1} \quad \textit{as required}$$

To minimise h the denominator must be maximised

Represent *sin(x) + cos(x) in the form kcos(x – a)*

sin(x) + cos(x) = kcos(x – a)
= kcos(x)cos(a) + ksin(x)sin(a)

equate coefficients



sin(x): *1 = ksin(a)*

cos(x): *1 = kcos(a)*

rearranging gives

tan(a) = 1 *since the solution lies in quadrant 1, a =* 45^0

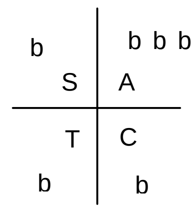
$k = \sqrt{2}$

$\sin(x) + \cos(x) + 1 = \sqrt{2}\cos(x - 45)^0 + 1$ and the max value of this is: $\sqrt{2} + 1$

so the minimum value of h is: $\frac{20}{\sqrt{2}+1} = 8 \cdot 28$

5. $2\sin 30t + \cos 30t = k\cos(30t - a)$
$= k\cos(x)\cos(a) + k\sin(x)\sin(a)$

equate coefficients



$\sin(30t)$: $2 = k\sin(a)$

$\cos(30t)$: $1 = k\cos(a)$

rearranging gives

$\tan(a) = 2$ since the solution lies in quadrant 1, $a = 63.4^4$

$k = \sqrt{5}$

$h = 20\sqrt{5}\cos(30t - 63.4)$

High tide occurs when $\cos(30t - 63.4)$ is a maximum value

i.e. $(30t - 63.4) = 0, 360$

$30t = 63.4, \ 423.4$

$t = 2.11, 14.11$ hours (the next high tide will be at 26.11 hours which is into day 2)

2.11 hours is 2 hours and 7 minutes
14.11 hours is 14 hours and 7 minutes

So high tide occurs at 02:07 hours and 14:07 hours

Low tide occurs when $\cos(30t - 63.4)$ is a minimum value

i.e. $(30t - 63.4) = 180, 540$

$30t = 243.3, 603.4$

$t = 8.11, 20.11$ hours (any other solutions are into the next day)

8.11 hours is 8 hours and 7 minutes
20.11 hours is 20 hours and 7 minutes

Low tide occurs at 08:07 hours and 20:07 hours