Further Calculus2 - Solutions

1. $\int_0^4 \sqrt{(4-x)}dx = \int_0^4 (4-x)^{1/2}dx = -\frac{2}{3}\left[(4-x)^{3/2}\right]_0^4$

$0 - (-\frac{2}{3}8) = 5\frac{1}{3}$

2. $\int_0^{p/4} sin(2x)dx = \left[-\frac{1}{2}cos(2x)\right]_0^{p/4}$

$= (-1/2 \cos(\frac{p}{2})) + (1/2 \cos(0))$

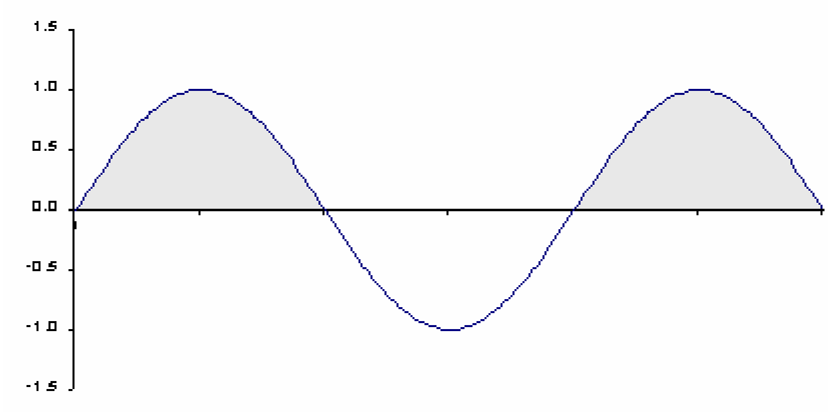
$= 1$

3. $\int_0^{p/4} sin(4x) + cos(4x)dx = \left[-\frac{1}{4}cos(4x) + \frac{1}{4}sin(4x)\right]_0^{p/4}$

(-1/4 + 0) - (-1/4 + 0) = 1/2

4.

First sketch the graph of $y = sin(3x)$ from x = 0 to x = p



To find the shaded area first find the limits i.e. where the graph crosses the x-axis.

sin(3x) = 0

3x = 0, p, 2p, 3p

x = 0, $p/3$, $2p/3$, p

The first shaded area is = $\int_0^{\frac{p}{3}} sin(3x)dx$

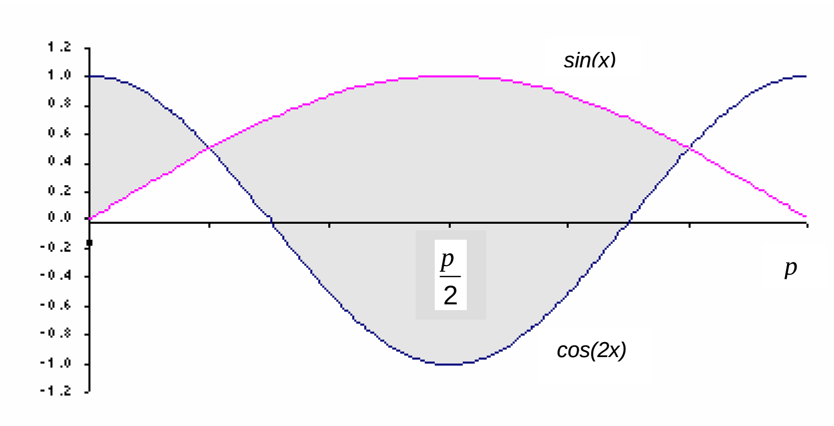
= -1/3 cos(p) + 1/3 cos(0)

= 2/3

The second shaded area between $2p/3$ and p is the same

so the total shaded area is 4/3 units^2

5. To find the shaded area, first find the limits. To do this, equate the two curves.



cos(2x) = sin(x)
1 – 2sin²(x) = sin(x)
2sin²(x) + sin(x) –1 = 0
(2sin(x) – 1)(sin(x) + 1) = 0
sin(x) = 1/2 and sin(x) = -1

$x = \frac{p}{6}, \ \frac{5p}{6}$ *any other solutions lie outside the given domain for x*

For the first area between 0 and $\frac{p}{6}$*, the upper curve is cos(2x) and the lower curve is sin(x)*

$$area = \int_0^{\frac{p}{6}} cos(2x) - sin(x)dx \quad = \quad \left[\frac{1}{2}sin(2x)+cos(x)\right]_0^{\frac{p}{6}}$$

$$= \quad \frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}-1 \quad = \frac{3\sqrt{3}}{4}-1 \ units^2$$