Course Revision Set2 Sols

1. $y - x = -10$

Subs $y = x - 10$

into $x^2 + y^2 + 4x + 6y - 40 = 0$

$x^2 + (x - 10)^2 + 4x + 6(x - 10) - 40 = 0$

$2x^2 - 10x = 0$

$x(x - 5) = 0$

$x = 0, x = 5$

when $x = 0, y = -10$
when $x = 5, y = -5$

*Points of intersection are A(0, -10) and B(5, -5)*

$AB = \sqrt{(5-0)^2 + (-5-(-10))^2} = \sqrt{50} = 5\sqrt{2}$

2. A(5, 7, -5), B(4, 7, -3) and C(2, 7, -4)

$\overrightarrow{AB} = (-1, 0, 2)$ $\qquad |\overrightarrow{AB}| = \sqrt{5}$

$\overrightarrow{AC} = (-3, 0, 1)$ $\qquad |\overrightarrow{AC}| = \sqrt{10}$

$\overrightarrow{BC} = (-2, 0, 1)$ $\qquad |\overrightarrow{BC}| = \sqrt{5}$

It can be seen that $|\overrightarrow{AC}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2$

so by the converse of Pythagoras ABC is right angled at B

By the scalar product:

$\overrightarrow{BA} = (1, 0, -2)$

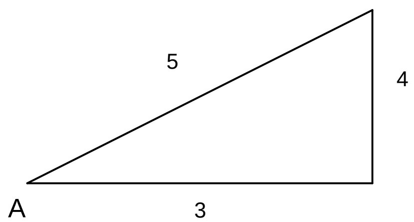
$\overrightarrow{BC} = (-2, 0, 1)$

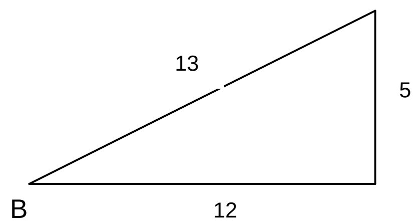
$\overrightarrow{BA} . \overrightarrow{BC} = -2 + 0 + 2 = 0$

since the dot product (or scalar product) = 0, angle ABC = $90^0$

3. sinA = 4/5 and tanB = 5/12 show that the exact value of sin(A + B) = 63/65 and tan(A – B) =33/56

First draw two right angled triangles and fill in the missing sides




sinA = 4/5 cosA = 3/5 sinB = 5/13 cosB = 12/13

sin(A + B) = sinAcosB + cosAsinB

$$= \frac{4}{5} x \frac{12}{13} + \frac{3}{5} x \frac{5}{13} = \frac{63}{65}$$ as required

tan(A – B) = $\frac{sin(A-B)}{cos(A-B)}$

sin(A – B) = sinAcosB - cosAsinB

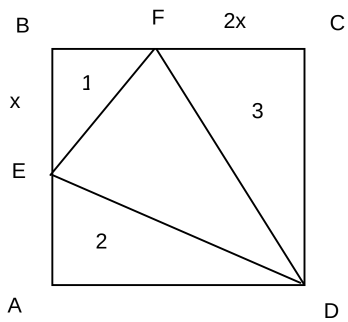
$$\frac{4}{5} x \frac{12}{13} - \frac{3}{5} x \frac{5}{13} = \frac{33}{65}$$

cos(A – B) = cosAcosB + sinAsinB

$$\frac{3}{5} x \frac{12}{13} + \frac{4}{5} x \frac{5}{13} = \frac{56}{65}$$

$$\frac{sin(A-B)}{cos(A-B)} = \frac{33}{65} \div \frac{56}{65} = \frac{33}{56}$$ as required

4.




Square side 10 so area of ABCD is 100cm$^2$

To find he area of DEF find areas 1, 2 and 3 and subtract them from 100

area 1 = $\frac{1}{2}(x)(10-2x) = 5x - x^2$

area 2 = $\frac{1}{2}(10)(10-x) = 50 - 5x$

area 3 = $\frac{1}{2}(2x)(10) = 10x$

area DEF = 100 – (5x – x$^2$ + 50 – 5x+ 10x)

= 50 - 10x + x$^2$ as required

A minimum value occurs at any minimum T.P. and this is a S.P.

S.Ps occur when dy/dx = 0 (let the area be y)

dy/dx = -10 + 2x = 0 (for SPs)

x = 5

To determine the nature of this SP examine dy/dx around x = 5

| x | $\rightarrow$ | 5 | $\rightarrow$ |
|---|---|---|---|
| dy/dx | - | 0 | + |

there is a minimum T.P. at (5, 25)

The minimum area of DEF is $25cm^2$

*5a* $2cos^2(x) = 1 \cdot 5$

$cos^2(x) = 3/4$

$x = \pm\frac{\sqrt{3}}{2}$

$x = 30^0, 150^0, 210^0, 330^0$

5*b* $sin(2x) = 2cos^2(x)$

$2sin(x)cos(x) = 2cos^2(x)$

$2cos^2(x) - 2sin(x)cos(x) = 0$

$2cos(x)[cos(x) – sin(x)] = 0$

$2cos(x) = 0$ *and* $cos(x) – sin(x) = 0$

$x = 90^0, 270^0$ $\quad$ $tan(x) = 1$
$x = 45^0, 225^0$

*solution:* $x = 45^0, 90^0, 225^0, 270^0$