Course Revision Set1 Sols

1. Let $G_n$ be the number of cars held at Glasgow
Let $E_n$ be the number of cars held at Edinburgh

$G_n + E_n = 3000$

$G_{n+1} = 0.8G_n + 0.4 E_n$

$G_{n+1} = 0.8G_n + 0.4(3000 - G_n)$

$G_{n+1} = 0.8G_n + 1200 - 0.4G_n$

$G_{n+1} = 0.4G_n + 1200$

This has a limit: $L = \frac{b}{1-a} = \frac{1200}{0.6} = 2000$

This means that in the long run there will be 2000 cars held at the Glasgow depot.

Check:

$E_{n+1} = 0.6E_n + 0.2G_n$

$E_{n+1} = 0.6E_n + 0.2(3000 - E_n)$

$E_{n+1} = 0.4E_n + 600$

This has a limit: $L = \frac{b}{1-a} = \frac{600}{0.6} = 1000$

This means that in the long run there will be 1000 cars held at the Edinburgh depot

2. A(-1,2,7), B(1,-2,5) and C(2,-4,4)

$\overrightarrow{AB} = (2,-4,-2)$ $\overrightarrow{BC} = (1, -2, -1)$

since $\overrightarrow{AB} = 2\overrightarrow{BC}$ and B is a common point, A, B and C are collinear, with B dividing AC in the ratio 2:1

3. centre (3,-3) and radius $\sqrt{5}$

equation: $x^2 + y^2 - 6x + 6y + 13 = 0$

equation of line: $x = 2 - 2y$ *substituting this into the circle equation gives:*

$(2 - 2y)^2 + y^2 - 6(2 - 2y) + 6y + 13 = 0$

$5y^2 + 10y + 5 = 0$

$5(y + 1)^2 = 0$ *double root implies the line is a tangent*
*(could also have used $b^2 - 4ac = 0$)*

*when $y = -1$*

$x = 2 - 2y = 4$

*point of contact is (4, -1)*

4. $y = x^3 + x^2 - x + 2$

*to find the roots of this polynomial factorise – try factors of 2*

| | | | | |
|---|---|---|---|---|
| -2 | 1 | 1 | -1 | 2 |
| | | -2 | 2 | -2 |
| | 1 | -1 | 1 | 0 |

*since remainder is 0, (x + 2) is a factor*

$y = (x + 2)(x^2 - x + 1)$ *note that this quadratic has no real roots, so the polynomial crosses the x-axis at one point only, when $x = -2$*

*y intercept occurs when $x = 0$ i.e. $y = 2$ (0, 2)*

*S.Ps occur when derivative = 0*

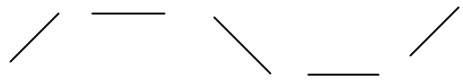
$$\frac{dy}{dx} = 3x^2 + 2x - 1 = 0$$

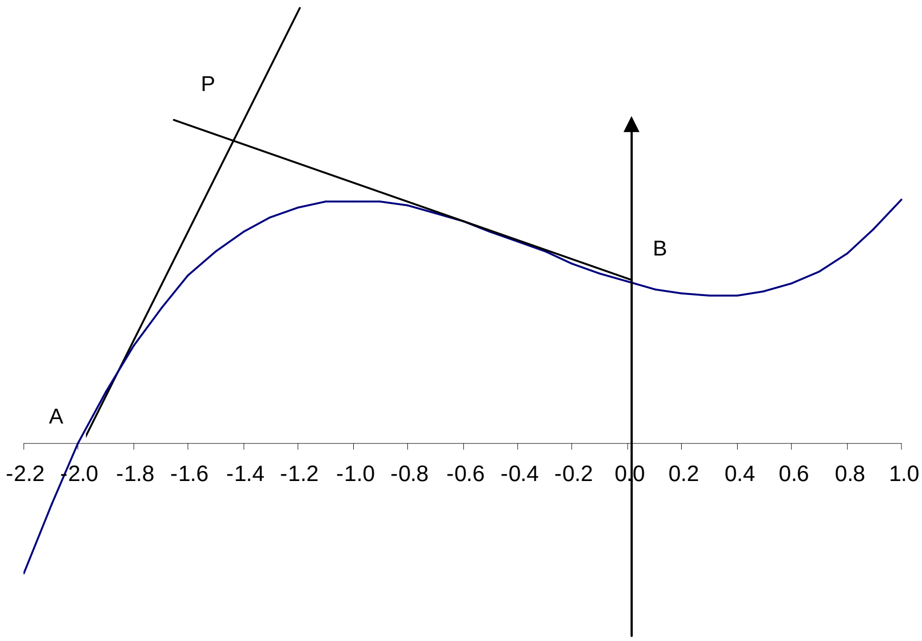
$(3x - 1)(x + 1) = 0$

$x = 1/3$ and $x = -1$

*to determine the nature of these S.Ps, examine dy/dx around $x = 1/3$ and $x = -1$*

| $x$ | $\rightarrow$ | $-1$ | $\rightarrow$ | $1/3$ | $\rightarrow$ |
|---|---|---|---|---|---|
| $dy/dx$ | + | 0 | - | 0 | + |






$dy/dx = 3x^2 + 2x - 1$ A is the point (-2, 0)

at point A, x= -2

gradient of tangent at A is: $3(-2)^2 + 2(-2) - 1 = 7$

Equation of tangent at A is *y = 7x + 14*

B is the point (0,2)

gradient of tangent at B is: $3(0)^2 + 2(0) - 1 = -1$

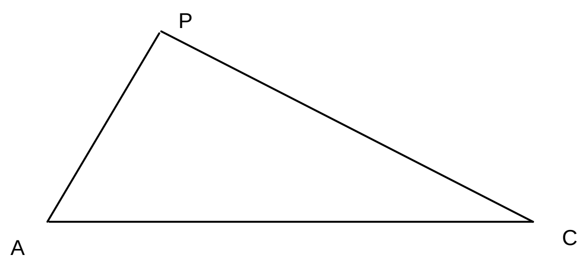
Equation of tangent at B is: *y = -x + 2*

Use simultaneous equations to find P(-3/2, 7/2)

To find angle APB use the gradients: Let C be the point on the x-axis where the tangent PB crosses

gradient of tangent PB is –1 so this makes an angle of $135^0$ with the x-axis, so PCA = $45^0$
gradient of tangent PA is 7 so this makes an angle of $81.8^0$ with the x-axis, so PAC = $81.8^0$

Angle APC = $53.2^0$




5. A(-4,-2), B(-4,4), C(6,4) and D(6,-2) is a rectangle

If all the points lie on the circumference, then the diagonal AC and the diagonal BD must be diameters of the circle.
The diagonals bisect, so the point of intersection is the centre of the circle.

To find the centre find the midpoint of AC

Midpoint AC = $\frac{1}{2}(-4+6, -2+4)$ = (1,1) Let the centre be the point E(1,1)

The radius is the distance from O to any point on the circumference (say A)

EA = $\sqrt{(-4-1)^2+(-2-1)^2}$ = $\sqrt{34}$

Equation of circle:

$(x-1)^2 + (y-1)^2 = 34$