Unit 2 Revision Exercise 5

The Circle   Solutions

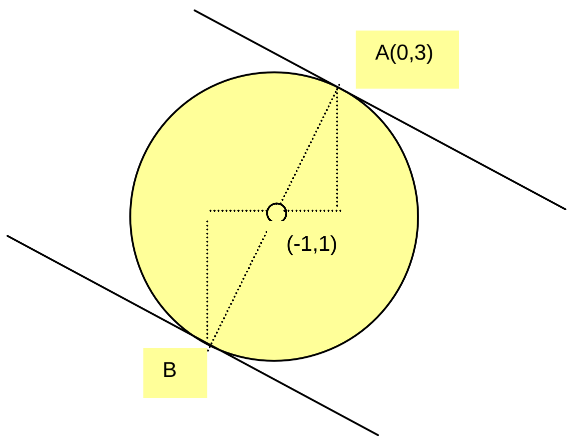
1. $x^2 + y^2 + 2x - 2y - 3 = 0$ *point on circle A(0,3)*

centre (-1,1)   *gradient of radius to A =* $\frac{3-1}{0-(-1)} = 2$

gradient of tangent at A = $-\frac{1}{2}$

equation of tangent at A: $y - 3 = -\frac{1}{2}x$

$\underline{2y + x = 3}$



The parallel tangent touches the other side of the circle at the point B(-2,-1)

Equation of parallel tangent at B is

$y - (-1) = -\frac{1}{2}(x - (-2))$

$2y + 2 = -x - 2$

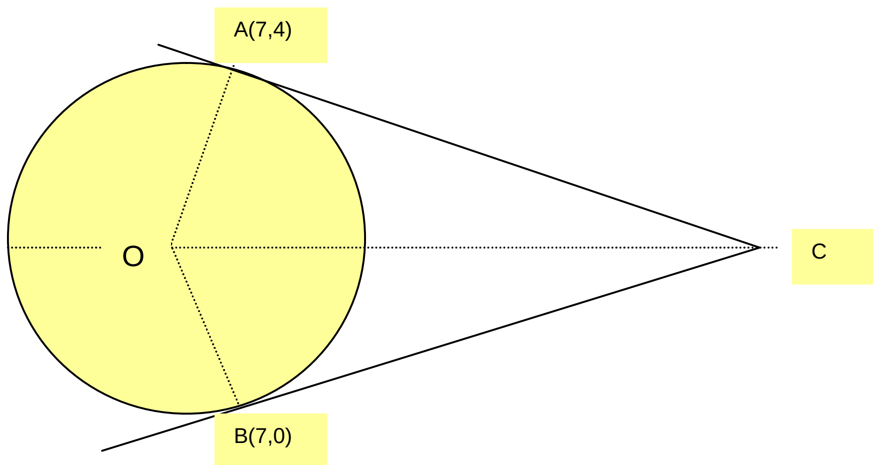
$2y + x = -4$

2. Centre at (4,2) Point on circumference (7,4)

radius = $\sqrt{(7-4)^2+(4-2)^2} = \sqrt{13}$ by the distance formula

Equation of circle

$x^2 + y^2 - 8x - 4y + 7 = 0$



$m_{OC} = \frac{4-2}{7-4} = \frac{2}{3}$ $\Rightarrow$ $m_{AC} = -\frac{3}{2}$ Equation of AC is: $2y + 3x = 29$

$m_{OC} = \frac{0-2}{7-4} = -\frac{2}{3}$ $\Rightarrow$ $m_{AC} = \frac{3}{2}$ Equation of BC is: $2y - 3x = -21$

Point of intersection $C(\frac{25}{3}, 2)$

One way to find the area of triangle ABC is to use $\frac{1}{2}ab\sin(C)$

The angle AC makes with OC = 180 - tan($-\frac{3}{2}$) = $56 \cdot 3^o$

So angle ACB = $112 \cdot 6^o$

To AC use the distance formula

AC = $\sqrt{(2)^2 + (\frac{4}{3})^2}$ = $2 \cdot 4$ AC = AB

Area of ABC = $\frac{1}{2} x 2 \cdot 4 x 2 \cdot 4 \sin(112 \cdot 6)$ = $2 \cdot 66 units^2$

3. Radius r touches the y – axis at *(0,3)*

Let the centre be (a,b) radius r equation $(x-a)^2 + (y-b)^2 = r^2$

since the circle touches the y – axis this means that x = 0 is a tangent.

Subs. x = 0 into the equation of the circle

$(0-a)^2 + (y - b)^2 = r^2$

$a^2 + y^2 - 2by + b^2 = r^2$

$y^2 - 2by + (a^2 + b^2 - r^2) = 0$

Since the line x = 0 is a tangent, this quadratic must have equal roots. So:

$(-2b)^2 - 4(a^2 + b^2 - r^2) = 0$

$4b^2 - 4a^2 - 4b^2 + 4r^2 = 0$

$4r^2 = 4a^2$

$r = a$

To find b, subs (0,30 into equation giving

$(0 - a)^2 + (3 - b)^2 = r^2$

since a = r then $(3 - b) = 0$ *i.e.* $b = 3$

Equation: $(x - r)^2 + (y - 3)^2 = r^2$ *as required*

subs the point (2,7) gives

$(2-r)^2 + (7-4)^2 = r^2$

$4 - 4r + r^2 + 16 = r^2$

$4r = 20$

$r = 5$ *radius 5, centre (5,3)*

equation

$$x^2 + y^2 - 10x - 6y + 9 = 0$$

On the x – axis, y= 0

$$x^2 - 10x + 9 = 0$$
$$(x-1)(x-9) = 0$$
$x = 1, x = 9$ *Distance between roots 8 units. (i.e. length of chord)*

4. $x^2 + (mx - 5m)^2 = 16$

$$x^2 + m^2x^2 - 10m^2x + 25m^2 - 16 = 0$$

$$(1 + m^2)x^2 - 10m^2 + (25m^2 - 16) = 0$$

equal roots $\Rightarrow b^2 - 4ac = 0$

$$(-10m^2)^2 - 4(1 + m^2)(25m^2 - 16) = 0$$

$$100m^4 - 4(25m^4 + 9m^2 - 16) = 0$$

$$64 - 36m^2 = 0$$

$m^2 = \frac{64}{36}$ $\quad$ $m = \pm\frac{8}{6} = \pm\frac{4}{3}$

Equation of tangents: $y = mx + c$

For $m = \frac{4}{3}$ *point (5,0)*

$$0 = \frac{4}{3}(5) + c$$

$$c = -\frac{20}{3}$$

equation of tangent

$$y = \frac{4}{3}x - \frac{20}{3}$$

$$3y = 4x - 20$$

For $m = -\frac{4}{3}$ *point (5,0)*

$$0 = -\frac{4}{3}(5) + c$$

$$c = \frac{20}{3}$$

equation of tangent

$$y = -\frac{4}{3}x + \frac{20}{3}$$

$$3y = 20 - 4x$$

5. *centre (4,3) radius 5*

equation $x^2 + y^2 - 8x - 6y = 0$ *(note* $5^2 = 4^2 + 3^2 - 0$*)*

To find the points of intersection with the x – axis y = 0

$x^2 - 8x = 0$
$x(x-8) = 0$
$x = 0, 8$ *P(0,0) and S(8,0)*

To find the points of intersection with the y – axis x = 0

$y^2 - 6y = 0$
$y(y-6) = 0$
$y = 0, 6$ *P(0,0) and Q(0,6)*

$m_{CP} = \frac{3-0}{4-0} = \frac{3}{4} \Rightarrow$ *gradient of tangent at P =* $-\frac{4}{3}$ *equation of tangent* $3y + 4x = 0$

$m_{CQ} = \frac{3-6}{4-0} = -\frac{3}{4} \Rightarrow$ *gradient of tangent at Q =* $\frac{4}{3}$ *equation of tangent* $y - 6 = \frac{4}{3}(x-0)$

$3y = 4x + 18$

$m_{CS} = \frac{3-0}{4-8} = -\frac{3}{4} \Rightarrow$ *gradient of tangent at S =* $\frac{4}{3}$ *equation of tangent* $y - 0 = \frac{4}{3}(x - 8)$

$3y = 4x - 32$

The tangents at Q and at S are parallel as their gradients are the same.

The fourth point is an equal distance from the centre as P

i.e. P(0,0) to C(4,3) to R(8,6)