Solutions – Exercise 3

6. $y = 12x^{\frac{1}{3}} - \frac{24}{\sqrt[3]{x}} + 2$

$y = 12x^{\frac{1}{3}} - 24x^{-\frac{1}{3}} + 2$

$\frac{dy}{dx} = 4x^{-\frac{2}{3}} + 8x^{-\frac{4}{3}}$ at $x = 8$ $\frac{dy}{dx} = \frac{4}{4} + \frac{8}{16} = \frac{3}{2}$

at $x = 8$ $y = 12(2) - \frac{24}{2} + 2 = 14$ point (8,14)

To find the equation of the tangent at $x = 8$ use (8,14) and $m = \frac{3}{2}$

$$y - b = m(x - a)$$
$$y - 14 = \frac{3}{2}(x - 8)$$
$$2y - 28 = 3x - 24$$
$$2y - 3x = 4$$

7.

For Killall weedkiller

$U_{n+1} = 0 \cdot 2U_n + 30$ (since 80% are killed this leaves 20%)

$L = \frac{b}{1-a} = \frac{30}{0 \cdot 8} = 37 \cdot 5$

(there will be $37 \cdot 5$ weeds in the long run using Killall)

For Killem weedkiller

$U_{n+1} = 0 \cdot 3U_n + 25$ (since 70% are killed this leaves 30%)

$L = \frac{b}{1-a} = \frac{25}{0 \cdot 7} = 35 \cdot 7$

8. $y = (x+1)^2(x-2)$ Roots at $x = -1, 2$ (-1,0) and (2,0)

y intercept occurs when *x = 0* i.e. $y = (1)^2(-2) = -2$ (0,-2)

Stationary points occur when $\frac{dy}{dx} = 0$

$$y = (x-2)(x^2 + 2x + 1)$$
$$y = x^3 - 3x - 2$$

$$\frac{dy}{dx} = 3x^2 - 3 = 0$$
$$x = \pm 1$$

To determine the nature of the two S.P.'s examine $\frac{dy}{dx}$ around $x = \pm 1$

	→	-1	→	1	→
$\frac{dy}{dx} = 3x^2 - 3$	+	0	-	0	+
	/	—	\	—	/

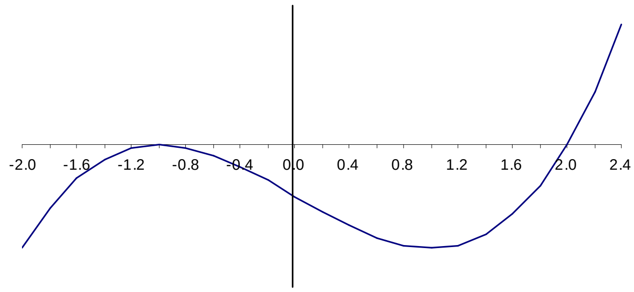
To find the value of the y coordinates, substitute $x = \pm 1$ into the original function.

Max T.P $x = -1$ $y = (-1)^3 - 3(-1) - 2 = 0$ max T.P. (-1,0)

Min T.P $x = 1$ $y = (1)^3 - 3(1) - 2 = -4$ min T.P. (1,-4)

$$y = x^3 - 3x - 2$$

As $x \to +\infty$, $y \to +\infty$ as $x \to -\infty$, $y \to -\infty$



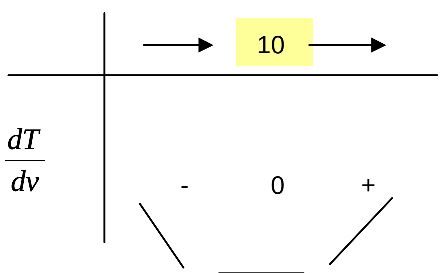
9. $T = \frac{D}{S} = \frac{5000}{v}\,hours$

Each hour $(1 + 0\cdot 0005v^3)$ tonnes of fuel are used.

Total amount of fuel used $= \frac{5000}{v}(1 + 0\cdot 0005v^3) = (\frac{5000}{v} + 2.5v^2)$ as required.

$\frac{dT}{dv} = -\frac{5000}{v^2} + 5v = 0$ $\quad v^3 = 1000$ so $\quad v = 10$

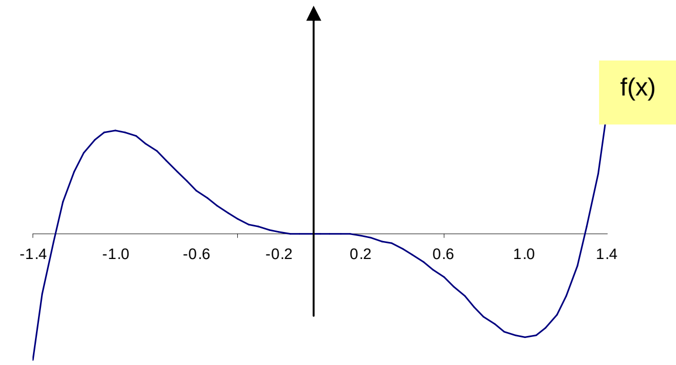
To determine the nature of this S.P. examine $\frac{dT}{dv}$ around $v = 10$

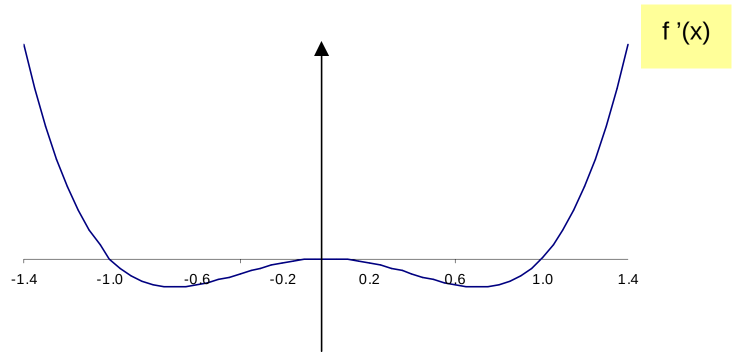


a minimum T.P. when $v = 10$

at this speed the total amount of fuel used is $\frac{5000}{10}(1 + 0\cdot 5) = 750$ tonnes

10.





The max. and min. values occur at any S.P.'s inside the interval, or at then end points of the interval.

S.P.'s occur when $\frac{dy}{dx} = 0$

$$y = 3x^5 - 5x^3$$

$$\frac{dy}{dx} = 15x^4 - 15x^2 = 0$$

$$15x^2(x^2 - 1) = 0$$

$$x = 0, \pm 1$$

When $x = -1$ $y = 3(-1)-5(-1) = 2$

$x = 0$ $y = 0$

$x = 1$ $y = (1) - 5(1) = -2$ Max. value = 2, min. value = -2