Solutions   Exercise 2

First draw a rough sketch of the quadrilateral.

1. $M_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{3-(-3)} = \frac{3}{6} = \frac{1}{2}$

$M_{DC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{5-(-1)} = \frac{3}{6} = \frac{1}{2}$

As $M_{AB} = M_{CD} \Rightarrow$ AB and CD are parallel

$M_{AD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{-1-(-3)} = -\frac{3}{2}$

$M_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{5-3} = -\frac{3}{2}$

As $M_{AD} = M_{BC} \Rightarrow$ AD and BC are parallel

We have a quadrilateral with two pairs of parallel lines – must show that it is not a square, rectangle or rhombus.

Since $M_{AB} x M_{AD} = \frac{1}{2} x - \frac{3}{2} \neq -1$ the lines AB and AD are not perpendicular – so ABCD cannot be a square or a rectangle.

The diagonal of a rhombus bisect one another at right angles – so find the gradients and then the equations of the diagonals. If product of gradients ≠ -1 diagonals are not perpendicular.

$M_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{5-(-3)} = 0$ So Ac is a horizontal line with equation $y = 1$

$M_{BD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{3-(-1)} = \frac{6}{4} = \frac{3}{2}$

(AC and BD are not perpendicular – ABCD is a parallelogram. Could also have found lengths of sides)

To find the equation of BD use $y - b = m(x - a)$ with $M = \frac{3}{2}$ and the point (3,4)

$y - b = m(x - a)$

$y - 4 = \frac{3}{2}(x - 3)$

$2y - 8 = 3x - 9$

$2y - 3x = -1$

To find the point of intersection E subs $y = 1$ into

$2y - 3x = -1$

$2 - 3x = -1$

$x = 1$

The point of intersection of the diagonals is E(1,1)

2. Let the number of customers that company A has be A_n

Let the number of customers that company B has be B_n

So: $A_N + B_N = 300{,}000$

Since company A retains 80% of its customers and gains 30% of B's the recurrence relation is given by

$$A_{n+1} = 0\cdot 8A_n + 0\cdot 3B_n \qquad B_n = 300{,}000 - A_n$$
$$A_{n+1} = 0\cdot 8A_n + 0\cdot 3(300{,}000 - A_n)$$
$$A_{n+1} = 0\cdot 8A_n + 90{,}000 - 0\cdot 3A_n$$
$$A_{n+1} = 0\cdot 5A_n + 90{,}000$$

This is of the form $U_{n+1} = aU_n + b$ and since $-1<a<1$ a limit L exists.

$$L = \frac{b}{1-a} = \frac{90{,}000}{0\cdot 5} = 180{,}000$$

This means that company A will eventually have 180,000 customers.

Check:

Since company B retains 70% of its customers and gains 20% of A's the recurrence relation is given by

$$B_{n+1} = 0\cdot 7B_n + 0\cdot 2A_n$$
$$B_{n+1} = 0\cdot 7A_n + 0\cdot 2(300{,}000 - B_n)$$
$$B_{n+1} = 0\cdot 7A_n + 60{,}000 - 0\cdot 2A_n$$
$$B_{n+1} = 0\cdot 5A_n + 60{,}000$$

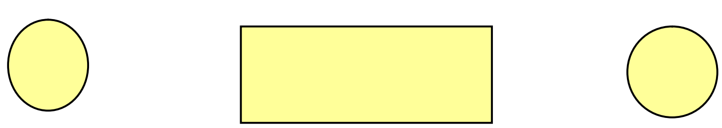
This is of the form $U_{n+1} = aU_n + b$ and since $-1<a<1$ a limit L exists.

$$L = \frac{b}{1-a} = \frac{60{,}000}{0\cdot 5} = 120{,}000$$

This means that company B will eventually have 120,000 customers.

Note that 180,000 + 120,000 = 300,000 as required.

3. The surface area is found by finding the area of the two circles (top and bottom) and the area of the side if the tin (which is a rectangle)



area circle = πr^2 area = $2\pi rh$ area = πr^2

Total surface area = $2\pi r^2 + 2\pi rh$

The volume is given by $V = \pi r^2 h$ so $h = \frac{V}{\pi r^2} = \frac{330}{\pi r^2}$

Substitute this into the formula for the Surface area to get

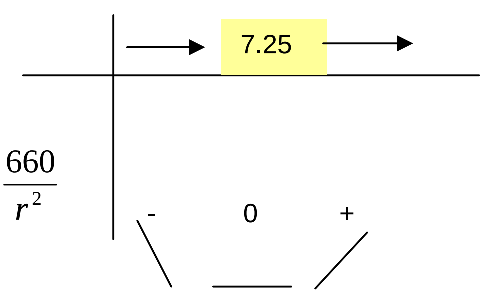
$$S = 2\pi r^2 + 2\pi r \frac{330}{\pi r^2} = 2\pi r^2 + \frac{660}{r} = 2\pi r^2 + 660 r^{-1}$$

Stationary points occur when $\frac{ds}{dr} = 0$ i.e. $\frac{ds}{dr} = 4\pi - \frac{660}{r^2} = 0$

$$r^2 = \frac{165}{\pi} \Rightarrow r \approx 7 \cdot 25$$

To determine the nature of this S.P. examine $\frac{ds}{dr}$ around $r = 7 \cdot 25$

$\frac{ds}{dr} = 4\pi - \frac{660}{r^2}$



There is a minimum T.P. at $(7 \cdot 25, 421.3)$

The minimum surface area occurs when $r = 7 \cdot 25$ cm and is 421.3cm^2

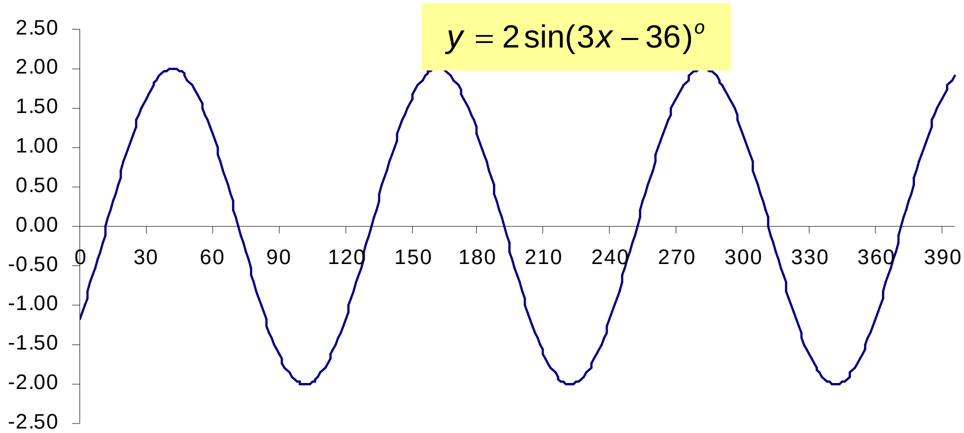
4. $y = 2\sin(3x - 36)^o$

The max value of this graph is 2 occurring when

$(3x - 36)^o = 90^o$ and $(3x - 36)^o = 450^o$

$3x = 126$ $3x = 486$

$x = 42^o$ $x = 162^o$



The min value of this graph is -2 occurring when

$(3x - 36)^o = 270^o$

$3x = 306$

$x = 102^o$

$2\sin(3x - 36)^o = 0\cdot 8$

$\sin(3x - 36) = 0\cdot 4$

$3x - 36 = 23\cdot 6^o, 156\cdot 4^o, 383\cdot 6^o,$

$3x = 59\cdot 6^o, 192\cdot 4^o, 419\cdot 6^o$

$x = 19\cdot 9^o, 64\cdot 1^o, 139\cdot 9^o$

all other solutions should be discarded as they lie outside the domain.

5. $U_{n+1} = aU_n + b$

$U_4 = aU_3 + b$

$U_3 = aU_2 + b$

$U_2 = 10$ Use back substitution to find a, b

$-1 = 10a + b$

$\underline{-5 \cdot 4 = -a + b}$

$11a = 4 \cdot 4$

$\underline{a = 0 \cdot 4}$

substituting to find b gives $10(0 \cdot 4) + b = 1$ $b = -5$

The recurrence relation is $U_{n+1} = 0 \cdot 4U_n - 5$

Since $U_2 = 10$ $10 = 0 \cdot 4U_1 - 5$ $U_1 = \frac{15}{0 \cdot 4} = 37 \cdot 5$

A limit exists as $-1 < a < 1$

limit $L = \frac{b}{1-a} = \frac{-5}{0 \cdot 6} = -8\frac{1}{3}$