Unit 1 Revision Exercise 1   Solutions

Find the points of intersection of the pairs of lines A and B and also A and C

1. $2x - y = 3$ (A) $\quad$ $2x - y = 3$ (A)

$\underline{3x + y = 2}$ (B) $\quad$ $\underline{4x - 5y = 9}$ (C)

$5x = 5$ $\quad$ $3y = -3$ $\quad$ 2(A) – (C)

$\underline{x = 1}$ $\quad$ $\underline{y = -1}$

$y = -1$ $\quad$ $x = 1$

check: 3-2=1 $\quad$ check: 4-(-5)=9

intersection of (A) and (B) is (1,-1) $\quad$ intersection of (A) and (C) is (1,-1)

$\Rightarrow$ lines A, B and C are concurrent and meet at point (1,-1)

2. a) $y - b = m(x-a)$ $\quad$ pts. A(7,3) B(10,-1)

$y - 1 = \frac{3}{4}(x+4)$ $\quad$ $M_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{10-7} = -\frac{4}{3}$

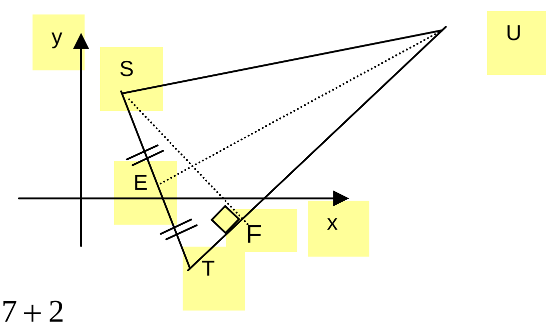
$4y - 4 = 3(x+4)$

$\underline{4y - 3x = 16}$ $\quad$ Equation AB: $y - 3 = -\frac{4}{3}(x-7)$

$\underline{3y + 4x = 37}$

$M_1 = \frac{3}{4}$ $\quad M_2 = -\frac{4}{3}$ $\quad$ since $M_1M_2 = -1$ the two lines are perpendicular.

3.



$M_{TU} = \frac{7+2}{13-4} = 1 \Rightarrow M_{SF} = -1$ as TU and SF are perpendicular.

Equation of SF : $y + x = 8$ $\quad$ using $y - b = m(x-a)$ with $m = -1$ and point (2,6)

Midpoint of ST is $(\frac{2+4}{2}, \frac{6-2}{2})$ So E is the point (3,2) $\quad M_{EU} = \frac{7-2}{13-3} = \frac{1}{2}$

Equation of EU is $2y - x = 1$ $\quad$ using $y - b = m(x-a)$ with $m = \frac{1}{2}$ and point (13,7)

To find the point of intersection use simultaneous equations:

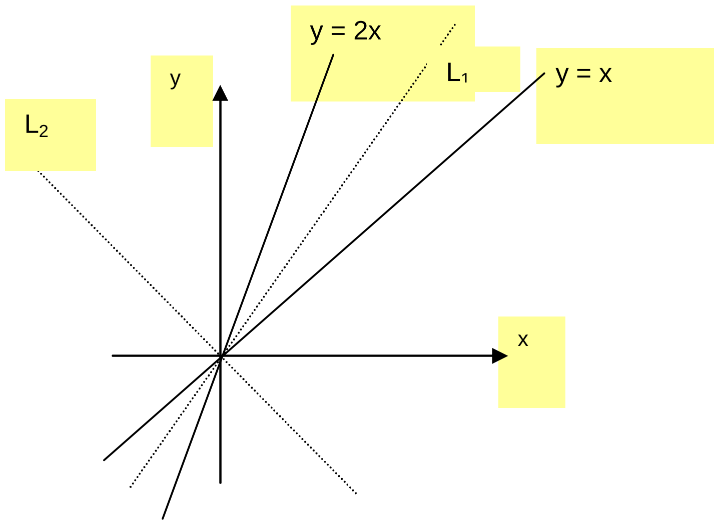
$y + x = 8$

$2y - x = 1$

$3y \quad = 9$

$y = 3$ and $x = 5$ (Check solution) Pt of intersection is (5,3)

4.



Let J_1 be the angle y = x makes with the x – axis. $\tan J_1 = 1$ so $J_1 = 45^o$

Let J_2 be the angle y = 2x makes with the x – axis. $\tan J_2 = 2$ so $J_2 = 63.4^o$

The angle which bisects J_1 and J_2 is 54.2^o so $m_{L1} = \tan 54.2^o = 1.39$

Equation of L_1 : $y = 1.39x$

Since L_1 and L_2 are perpendicular, $M_{L1} M_{L2} = -1$ So $M_{L2} = -0.72$

Equation of L_2 : $y = -0.72x$

5. Let $y = \dfrac{4-x}{x}$ $f^{-1}(f(x)) = f^{-1}(\dfrac{4-x}{x})$

xy = 4 – x

xy + x = 4

x(y + 1) = 4

$$x = \frac{4}{y+1} \quad \text{so } f^{-1}(x) = \frac{4}{x+1}$$

$$= \frac{4}{\frac{4-x}{x}+1}$$

$$= \frac{4}{\frac{4-x+x}{x}} = \frac{4x}{4} = x$$

as req[d]

$$f(f^{-1}(x)) = f\left(\frac{4}{x+1}\right)$$

$$= \frac{4-\frac{4}{x+1}}{\frac{4}{x+1}} = \frac{\frac{4x+4-4}{x+1}}{\frac{4}{x+1}} = \frac{4x}{4} = x$$ as req[d]

6. Let $y = \frac{3x-2}{2x}$

2xy = 3x –2

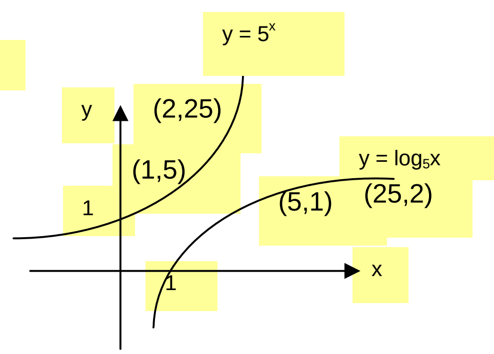
x(2y – 3) = -2

$$x = \frac{-2}{2y-3} \quad \text{so } g^{-1}(x) = \frac{-2}{2x-3}$$

$$g(g^{-1}(x)) = g\left(\frac{-2}{2x-3}\right)$$

$$g^{-1}(g(x) = g^{-1}\left(\frac{3x-2}{2x}\right)$$

7.



$$= \frac{3\left(\frac{-2}{2x-3)}\right)-2}{2\left(\frac{-2}{2x-3)}\right)}$$

$$= \frac{\frac{-6-4x+6}{2x-3}}{\frac{-4}{2x-3}}$$

$$= \frac{-4x}{-4} = x \text{ as req}^{d}$$

$$= \frac{-2}{2\left(\frac{3x-2}{2x}\right)-3}$$

$$= \frac{-2}{\frac{6x-4-6x}{2x}}$$

$$= \frac{-2(2x)}{-4} = x \text{ as req}^{d}$$

8 a) $y = a^x + b$ use the points (0,5)

$5 = a^0 + b$ since $a^0 = 1$ b = 4

next subst (2,13)

$13 = a^2 + 4$ $a^2 = 9$

a= 3

So $y = 3^x + 4$

b) $y = \log_a(x+b)$

using point (-4,0)

$\log_a(1) = 0$ so b = 5
since (-4+b)=1

using the point (20,2)
$\log_a 25 = 2$ so $a^2 = 25$ a = 5

$y = \log_5(x+5)$

$y = \log_a(x+b)$ we know that $\log_a 1 = 0$ so 3 + b =1 b = -2

also $\log_a a = 1$

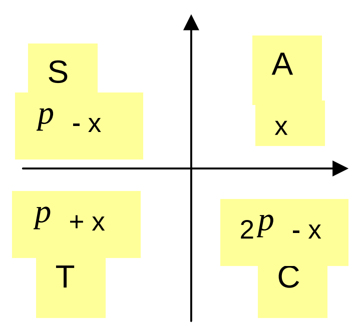
$\log_a(6-2) = 1$

$\log_a 4 = 1$ so a =4 $y = \log_4(x-2)$

9. a) $\sin\left(\frac{5p}{3}\right) = -\sin\left(\frac{p}{3}\right) = -\frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{7p}{6}\right) = -\cos\left(\frac{p}{6}\right) = -\frac{\sqrt{3}}{2}$

c) $\tan\left(\frac{11p}{6}\right) = \tan\left(\frac{p}{6}\right) = \frac{1}{\sqrt{3}}$



Find the related angle x in the first quadrant

d) $\cos\left(\frac{2p}{3}\right) = -\frac{1}{2}$

10. a) $2\sin(2x) = 1$

$\sin(2x) = \frac{1}{2}$

$(2x) = 30^o, 150^o, 390^o, 510^o$

$x = 15^o, 75^o, 195^o, 255^o$

b) $\sqrt{3}\tan(2x) - 1 = 0$

$\tan(2x) = \frac{1}{\sqrt{3}}$

$(2x) = 30^o, 210^o, 390^o, 570^o$

$x = 15^5, 105^o, 195^o, 285^o$

c) $2\cos(x) + \sqrt{3} = 0$

$\cos(x) = -\frac{\sqrt{3}}{2}$

$x = 150^o, 210^o$

11. a) $4\tan(2x) + 4 = 0$

$\tan(2x) = -1$

$(2x) = \frac{3p}{4}, \frac{7p}{4}, \frac{11p}{4}, \frac{15p}{4}$

$x = \frac{3p}{8}, \frac{7p}{8}, \frac{11p}{8}, \frac{15p}{8}$

b) $2\sin(3x) - 1 = 0$

$\sin(3x) = \frac{1}{2}$

$(3x) = \frac{p}{6}, \frac{5p}{6}, \frac{13p}{6}, \frac{17p}{6}, \frac{25p}{6}, \frac{29p}{6}$

$x = \frac{p}{18}, \frac{5p}{18}, \frac{13p}{18}, \frac{17p}{18}, \frac{25p}{18}, \frac{29p}{18}$

c) $4\cos(2x) - 2 = 0$

$\cos(2x) = \frac{1}{2}$

$(2x) = \frac{p}{3}, \frac{5p}{3}, \frac{7p}{3}, \frac{11p}{3}$

$x = \frac{p}{6}, \frac{5p}{6}, \frac{7p}{6}, \frac{11p}{6}$

Always check the given domain; if $0 \leq x \leq 2p$ then the solution must be in radians.