Solutions Exercise 3

21. $2x - y + 5 = 0$ and $4x - 2y - 7 = 0$.

$y = 2x + 5$ $\quad\quad$ $2y = 4x - 7$

$\quad\quad\quad\quad\quad\quad\quad$ $y = 2x - (7/2)$

Since both lines have the same gradient, they are parallel and so will never meet.

22.

f(x)	$x^2 + 1$	$-\frac{2x}{3}$	$\frac{x}{x+1}$	$\frac{2x^2-1}{x^2-1}$
f(3)	10	-2	3/4	17/8
f(-2)	5	4/3	2	7/3
f(1/2)	5/4	-1/3	1/3	2/3
domain	$x: x \in \Re$	$x: x \in \Re$	$x: x \in \Re\ x \neq -1$	$x: x \in \Re\ x \neq \pm 1$

23. P(0,4), Q(-2,-6) and R(6,-2)

$m_{QR} = \frac{4}{8} = \frac{1}{2}$ $\quad$ *m of alt to P = -2 Equation of alt to P* $y - 4 = -2(x-0)$ $\quad$ $y + 2x = 4$

$m_{PR} = \frac{-6}{6} = -1$ $\quad$ *m of perp bisector = 1 midpoint of PR = (3,1)*

equation of perp. bisector $\quad$ $y - 1 = (x - 3)$

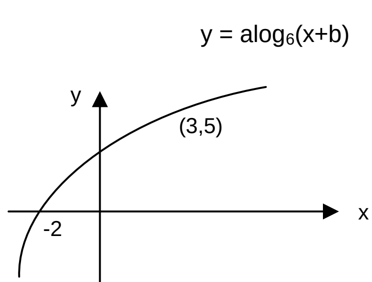
$y - x = -2$

Intersection of lines $\quad$ $y + 2x = 4$

$\underline{y - x = -2}$

$x = 2 \quad y = 0$ $\quad$ *Point of intersection (2,0)*

24.



a) b = 3

b) $y = a^x + b$

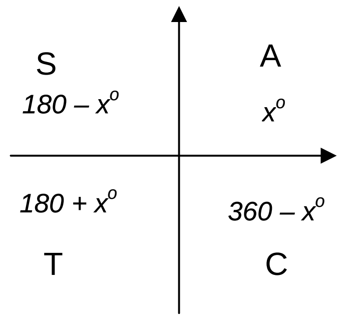
substituting gives
gives
$6 = a\log_6 6$
$a = 6$

substituting
$3 = a^0 + b$
$b = 2$
substituting
$11 = a^2$

next point gives
+ 2

$a = 2$

25. a) $\cos(2x) + \cos(x) = 0$
$2\cos^2(x) + \cos(x) - 1 = 0$
$(2\cos(x) - 1)(\cos(x) + 1) = 0$
$\cos(x) = \frac{1}{2}$ and $\cos(x) = -1$
$x = 60^o, 180^o, 300^o$



b) $5\cos(2x) + 7\sin(x) + 7 = 0$
$5(1 - 2\sin^2(x)) + 7\sin(x) + 7 = 0$
$12 + 7\sin(x) - 10\sin^2(x) = 0$
$10\sin^2(x) - 7\sin(x) - 12 = 0$
$(5\sin(x) + 4)(2\sin(x) - 3) = 0$

$\sin(x) = -\frac{4}{5}$, $\sin(x) = \frac{3}{2}$ (this solution is not possible)

$\sin(x) = -\frac{4}{5}$, the solution will lie in quadrant 3 and quadrant 4 as sin is negative.

related angle $\sin^{-1}\frac{4}{5} = 53.13^o$

Solution: $x = 233\cdot13^o,\ 306\cdot87^o$

26. $s = t^3 - 2t^2 + 3t$

$s(2) = 8 - 8 + 6 = 6m$
$s(1) = 1 - 2 + 3 = 2m$

Distance travelled between 1sec and 2sec was 4m

Average speed between 1sec and 2sec = $4ms^{-1}$

$\frac{ds}{dt} = 3t^2 - 4t + 3$ after 2 seconds $\frac{ds}{dt} = 12 - 8 + 3 = 7ms^{-1}$

27. a) $f(x) = 2x^2 - 10x$

$f'(x) = 4x - 10$

increasing when $f'(x) > 0$

i.e. $x > \frac{5}{2}$

b) $f(x) = x^3 - 3x^2 - 6$

$f'(x) = 3x^2 - 6x$

$f'(x) > 0$ when $3x^2 - 6x > 0$

$3x(x - 2) > 0$

this returns a positive value when $x < 0$ and $x > 2$

28. $y = x + \frac{1}{x}$. when $x = \frac{1}{2}$ $y = \frac{1}{2} + 2 = \frac{5}{2}$ so this point lies on the curve.

$y = x + x^{-1}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$ at $x = \frac{1}{2}$ $\frac{dy}{dx} = 1 - \frac{1}{1/4} = -3$ the gradient of the tangent at A is -3

Equation of tangent at A:

$y - \frac{5}{2} = -3(x - \frac{1}{2})$

$2y - 5 = -6x + 3$

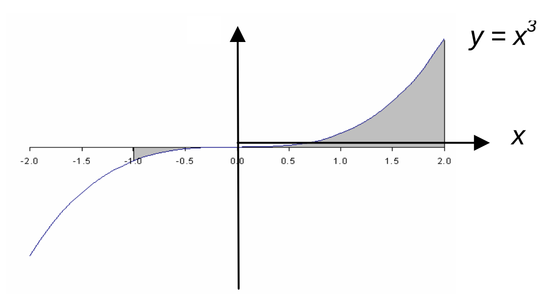
$2y = -6x + 8$

This line meets the x – axis at C when $y = 0$ $x = 4/3$ C(4/3,0)

This line meets the y – axis at D when $x = 0$ $y = 4$ D(0,4)

$CD^2 = \frac{16}{9} + 16 = \frac{16 + 144}{9} = \frac{160}{9}$ $CD = \sqrt{\frac{16x10}{9}} = \frac{4}{3}\sqrt{10}$ units as required

29.



Since part of the shaded area lies beneath the x – axis, the integration must be done in two parts.

$\int_{-1}^{0} x^3 dx = \left[\frac{x^4}{4}\right]_{-1}^{0} = 0 - \frac{1}{4} = -\frac{1}{4}$ since this lies beneath the x-axis, ignore minus sign.

$$\int_0^2 x^3 dx = \left[\frac{x^4}{4}\right]_0^2 = 4 - 0 = 4 \text{ units}$$

Total shaded area = $4\frac{1}{4} units^2$

30. *sin(x+30)°* = *sin(x), show that* *tan(x)°* = *2 +* $\sqrt{3}$

sin(x)cos(30) + cos(x)sin(30) = sin(x)

$$\frac{\sqrt{3}}{2}\sin(x) + \frac{1}{2}\cos(x) = \sin(x)$$

divide both sides by sin(x)

$$\frac{\sqrt{3}}{2} + \frac{\cos(x)}{2\sin(x)} = 1 \qquad \sqrt{3} + \frac{\cos(x)}{\sin(x)} = 2 \qquad \frac{\cos(x)}{\sin(x)} = 2 - \sqrt{3}$$

$$\frac{\sin(x)}{\cos(x)} = \frac{1}{2-\sqrt{3}} \qquad \tan(x) = \frac{1}{2-\sqrt{3}}$$

this is the same as 2 + $\sqrt{3}$ *which can be obtained by rationialising the denominator i.e.*

$$\frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

31. *f(x) = 3x + 1* and *g(x) = 4 –* x^2, find *f(g(x))* and *g(f(x))*

f(g(x))	*g(f(x))*
= $f(4 - x^2)$	= *g(3x + 1)*
= $3(4 - x^2) + 1$	= $4 - (3x + 1)^2$
= $13 - x^2$	= $4 - (9x^2 + 6x + 1)$
	= $3 - 6x - 9x^2$